# Complex power inverse problems

## ACM SIGENERGY Graduate Student Seminar Series

Samuel Talkington

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School of Electrical and Computer Engineering
Georgia Institute of Technology

Georgia Tech College of Engineering
School of Electrical
and Computer Engineering

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# Motivation and Background

# Inverse problems

### Inverse problem

Causal system or process

$$F : \mathcal{X} \mapsto \mathcal{Y}.$$

True causation $\boldsymbol{x}^*$, indirect output measurements $\boldsymbol{y}$, noise $\boldsymbol{\eta} \sim \mathcal{D}$

$$\boldsymbol{y} = F(\boldsymbol{x}^*) + \boldsymbol{\eta}$$

Can we recover the causation $\boldsymbol{x}^*$ of our measurements?

# Complex power physics

# Electric power is complex (literally)

An electric power network is an undirected graph

$$\mathcal{G} = (\mathcal{N}, \mathcal{V}).$$

Every node $i \in \mathcal{N}$ has a complex **voltage phasor** and **power injection**:

$$v_i \angle \theta_i \in \mathbb{C}, \qquad p_i + jq_i \in \mathbb{C}.$$

# Power factor and the power triangle

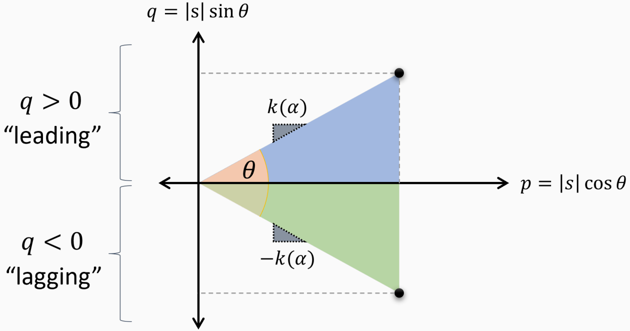


**Figure 1:** The power triangle relates complex power to the power angle

# Power factor and the power triangle

**The *power factor* $\alpha_i \in (0, 1] \subset \mathbb{R}$ at nodes $i \in \mathcal{N}$:**

Ratio of the real part (active power) of the complex power injection at $i \in \mathcal{N}$ to its magnitude:

$$\alpha_i = \cos(\text{atan2}(q_i, p_i)), \quad i \in \mathcal{N}, \tag{1}$$

where $\text{atan2}(\cdot, \cdot)$ is the two-argument arc tangent.

# Motivation 1: Missing measurements

# The case of the missing voltage phasor

Every node $i \in \mathcal{N}$ has a complex **voltage phasor** and **power injection**:

$$v_i \angle \theta_i \in \mathbb{C}, \quad p_i + j q_i \in \mathbb{C}.$$

It's **hard to find** the voltage phase angles in the real world.

# The case of the missing voltage phasor

Example: PJM, circa 2022

1. 400 PMUs throughout PJM territory[1]
2. Only required on substations

[1] PJM, "Synchrophasor Technology Roadmap", 2022.

# The case of the missing voltage phasor

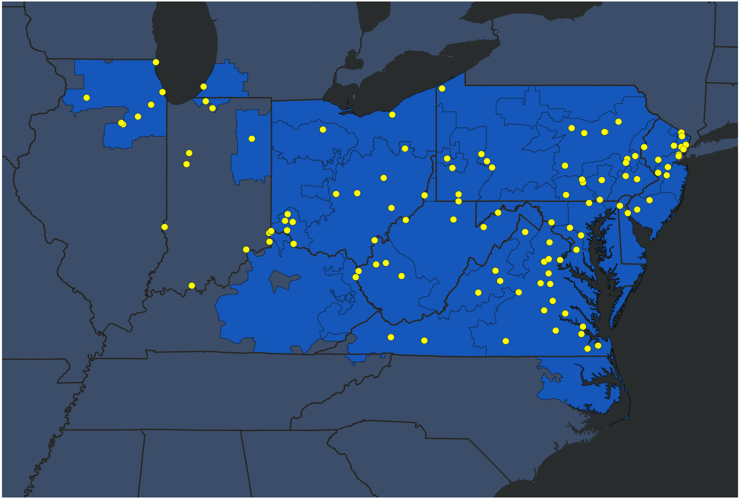

Figure 2: circa 2022 PMU deployment in PJM[2]

[2] Credit PJM, "Synchrophasor Technology Roadmap", 2022. Source: *https://www.pjm.com/markets-and-operations/ops-analysis/synchrophasor-technology*

# The case of the missing voltage phasor

$\approx$ 3000 phasor measurement units (PMUs) in North America [1].

Rare in:

1. Distribution systems
2. Transmission system boundaries
3. Rural transmission systems
4. Underserved areas

# Motivation 2: Missing models

# The case of the missing model

Challenge: realistic physical models in electric power systems.

Two types of models are often **unknown:**

1. **Network model:** Network topology parameters.
2. **Reactive power model:** Control law parameters.

# The case of the missing model

The network topology is encoded in the *admittance* matrix:

$$Y = G + jB \in \mathbb{C}^{n \times n}.$$

These parameters are **difficult to come by** in the real world.

# The case of the missing model

The non-linear power flow equations govern the power injections at every node $i \in \mathcal{N}$:

$$p_i = v_i \sum_{k=1}^{n} v_k (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}), \tag{2a}$$

$$q_i = v_i \sum_{k=1}^{n} v_k (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}). \tag{2b}$$

They are functions of the network topology.

# The case of the missing model

The causal factors for the reactive power injections:

$$\boldsymbol{q} = f(\,\cdot\,|\boldsymbol{\theta}),$$

are the *control law* parameters $\boldsymbol{\theta} \in \Theta$. These are often unknown, or can change over time.

It’s pretty hard to be a power engineer without this information!

What do we actually know?

# What do we actually know?

What do we **actually** know in practice?

Commonly, we receive measurements of the form [2]

$$v_i \in \mathbb{R}, \quad p_i + jq_i \in \mathbb{C}.$$

1. $v_i$ —voltage ***magnitude***
2. $p_i$ —active (real) power
3. $q_i$ —reactive (imaginary) power[3]

**No network model, either!**

[3] e.g., from a historical or chosen power factor [2]

# Reactive power inverse problems

# Problem 1: reactive power system identification

**Inverse problem: reactive power system identification**

What is the causation (control law parameters) of our reactive power measurements?

$$\boldsymbol{q} = F(\boldsymbol{\theta}^*) + \boldsymbol{\eta}$$

# Control paradigms

There are several reactive power control frameworks:

1. **Power factor control**: control the relationship between $p, q$.
2. **Volt-VAR control**: control the relationship between $q, v$.

# Graphical Depiction of Power Factor Control

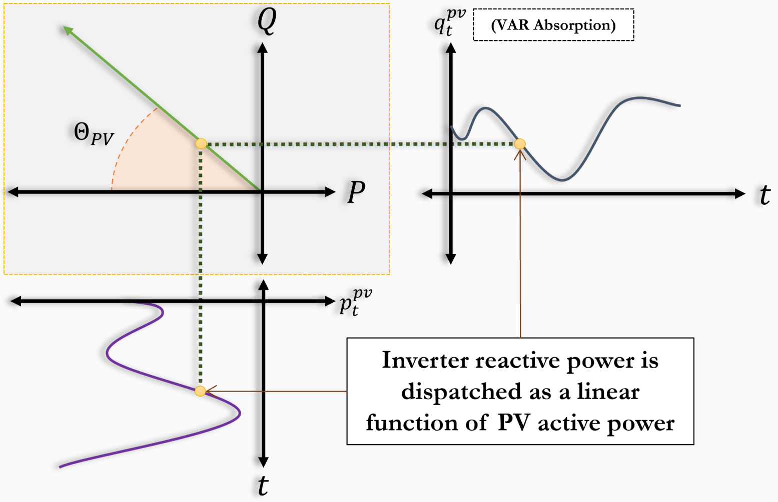


Figure 3: Illustration of real-time inverter power factor control [3].

# Graphical Depiction of Volt-VAR Control

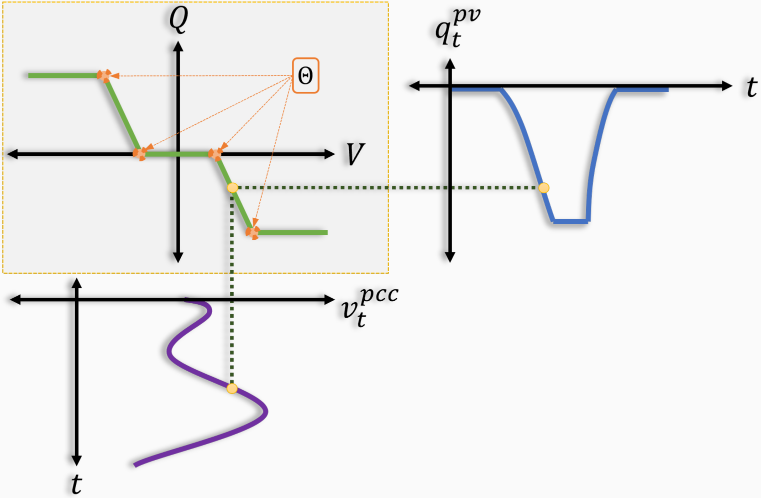


Figure 4: Illustration of real-time inverter volt-VAR control [3].

# Parameterized Fixed Power Factor Control

The reactive power injection of an inverter with power factor control is determined by a line in the complex plane:

$$q = \phi_{\Theta}(p) = \frac{\Delta q}{\Delta p} p \tag{3}$$

The slope of this line is the "sensitivity" of the IBR reactive power injections to real power injections.

# Parameterized Fixed Power Factor Control

Use trigonometry to relate the line slope to the power factor setting:

$$\text{pf} = \cos(\phi_V - \phi_I) \implies \text{pf} = \cos\left(\text{atan2}\left(\frac{\Delta q}{\Delta p}\right)\right), \tag{4}$$

where $\phi_V, \phi_I$ is the phase angle of the voltage and current, respectively.

# Parameterized Volt Var Control

For the response of a Volt-VAR control law, a general representation of the reactive power response is given by a parameterized function of PCC voltage $v_t^{pcc}$,

$$\phi_\Theta(v_t^{pcc}) := \begin{cases} Q_1 & v_t^{pcc} \leq V_1, \\ v_t^{pcc} \frac{Q_2 - Q_1}{V_2 - V_1} + b_1 & V_1 < v_t^{pcc} < V_2, \\ 0 & V_2 \leq v_t^{pcc} \leq V_3, \\ v_t^{pcc} \frac{Q_4 - Q_3}{V_4 - V_3} + b_2 & V_3 \leq v_t^{pcc} \leq V_4, \\ Q_4 & v_t^{pcc} > V_4. \end{cases} \tag{5}$$

where $b_1 = Q_1 - V_1 \frac{Q_2 - Q_1}{V_2 - V_1}$ and $b_2 = Q_3 - V_3 \frac{Q_4 - Q_3}{V_4 - V_3}$, and $\theta_i := [P_i, Q_i]^T$, $i = 1, \ldots, N$.

# Estimating reactive power parameters

$$\underset{\theta}{\text{minimize}} \; \ell(P, Q, V|\theta) \quad \text{subject to } \theta \in \Theta, \tag{6}$$

where $\Theta$ is the set of feasible parameters for the control mode.

Several possible loss functions can be used, such as MLE and regularized norm approximation losses.

# Fixed power factor system identification result summary

# Performance Summary

**Table 1:** Performance evaluations (MAE) of power factor estimation for 50 real BTM PV systems

| PF Control Type | $\ell_1$ | Huber, $\epsilon = 7 \times 10^{-2}$ |
|---|---|---|
| Unity | 0.0000571 | 0.00343 |
| Non-unity | 0.0104 | 0.0103 |

**Results:**

We can estimate **unity and non unity** PF control settings from net load data with high accuracy.

# Performance Summary

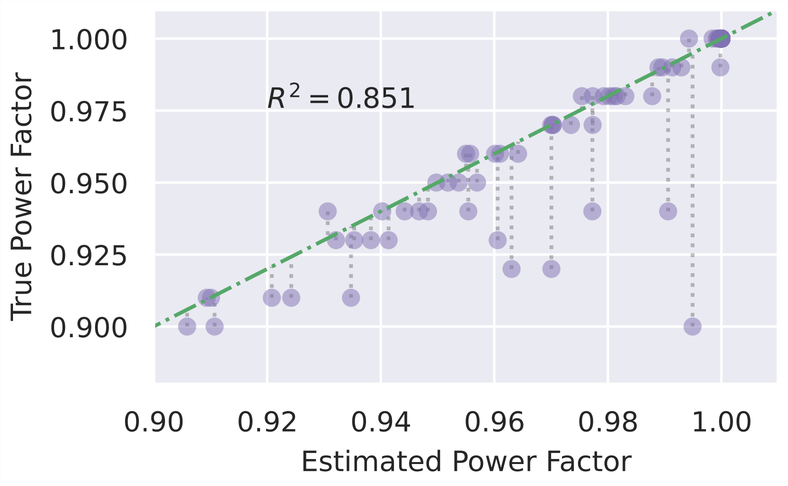


**Figure 5:** Scatter plot of estimated power factor vs. true power factor for all datasets studied

# Volt-VAR system identification result summary

# Performance summary

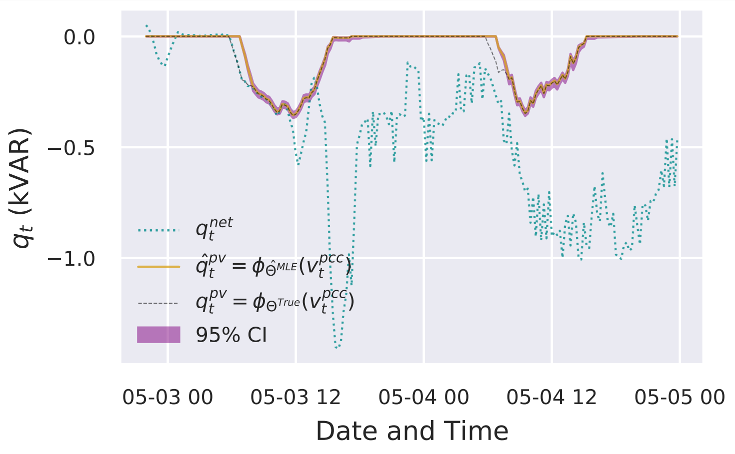


Figure 6: Reconstruction of VVC reactive power time series with estimated control law.

# References

S. Talkington, S. Grijalva, M. J. Reno, J. A. Azzolini, “Solar PV Inverter Reactive Power Disaggregation and Control Setting Estimation”, IEEE Transactions on Power Systems, 2022

# Phaseless inverse problems

# Voltage phase retrieval

**Inverse problem: voltage phase retrieval**

What are the causal factors (power-phase angle sensitivity matrices) of our complex power measurements?

$$\boldsymbol{p} + j\boldsymbol{q} = F\left(\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}}^*, \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{\theta}}^*\right) + \boldsymbol{\eta}$$

*Corollary:* what are the voltage phase angles?

# Case study: Newton-Raphson Power Flow model

How to solve the non-linear power flow equations?

Classic approach: *Newton-Raphson power flow.*

Iteratively solve a linear system of equations of the form

$$\begin{bmatrix} \Delta p \\ \Delta q \end{bmatrix} = \begin{bmatrix} \frac{\partial p}{\partial \boldsymbol{\theta}}(x) & \frac{\partial p}{\partial v}(x) \\ \frac{\partial q}{\partial \boldsymbol{\theta}}(x) & \frac{\partial q}{\partial v}(x) \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta} \\ \Delta v \end{bmatrix} = J(x)\Delta x. \tag{7}$$

1. $\Delta \boldsymbol{p}, \Delta \boldsymbol{q} \in \mathbb{R}^n$ are small perturbations in the active and reactive power injections
2. $\Delta \boldsymbol{\theta}, \Delta \boldsymbol{v} \in \mathbb{R}^n$ are small perturbations in the voltage phase angles and magnitudes

# Case study: Newton-Raphson Power Flow model

$$\begin{bmatrix} \Delta p \\ \Delta q \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial p}{\partial \boldsymbol{\theta}}(x) & \frac{\partial p}{\partial v}(x) \\ \frac{\partial q}{\partial \boldsymbol{\theta}}(x) & \frac{\partial q}{\partial v}(x) \end{bmatrix}}_{:=J(x)} \begin{bmatrix} \Delta \boldsymbol{\theta} \\ \Delta v \end{bmatrix} = J(x)\Delta x, \tag{8}$$

1. The matrix $J(x) \in \mathbb{R}^{2n \times 2n}$ is the power flow Jacobian matrix.
2. Derivatives of the power flow equations (2) with respect to the voltage magnitudes $\boldsymbol{v}$ and phase angles $\boldsymbol{\theta}$.

# Case study: Newton-Raphson Power Flow model

$$\begin{bmatrix} \Delta p \\ \Delta q \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial p}{\partial \boldsymbol{\theta}}(x) & \frac{\partial p}{\partial v}(x) \\ \frac{\partial q}{\partial \boldsymbol{\theta}}(x) & \frac{\partial q}{\partial v}(x) \end{bmatrix}}_{:= J(x)} \begin{bmatrix} \Delta \boldsymbol{\theta} \\ \Delta v \end{bmatrix} = J(x)\Delta x, \tag{9}$$

1. Can we learn this matrix as a proxy model?
2. Can we exploit the structure of this matrix?

# What is the problem?

1. How can we recover voltage phasors from their magnitudes?
2. How can we recover a *phaseless model* of the voltage phasors?
3. How can we do this provably?

# Solution overview

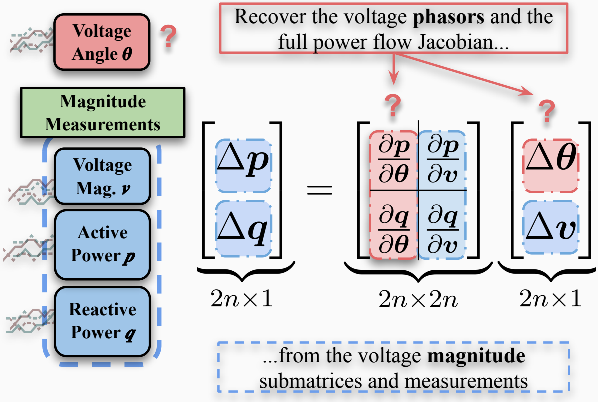


Figure 7: Voltage phasor recovery via power flow Jacobian recovery

# New characterization of Power Flow Jacobian structure

**Lemma 1: Phaseless power flow Jacobian structure**

We can write the partial derivatives of power injections with respect to phase angles without the phase angles and without the grid model.

$$\frac{\partial p_i}{\partial \theta_k} = \begin{cases} v_k \frac{\partial q_i}{\partial v_k} & i \neq k \\ v_i \frac{\partial q_i}{\partial v_i} - 2q_i & i = k, \end{cases} \tag{10a}$$

$$\frac{\partial q_i}{\partial \theta_k} = \begin{cases} -v_k \frac{\partial p_i}{\partial v_k} & i \neq k \\ -v_i \frac{\partial p_i}{\partial v_i} + 2p_i & i = k. \end{cases} \tag{10b}$$

These are functions of the **power injection measurements!**

# New characterization of the structure of the Power Flow Jacobian

The power-voltage phase angle sensitivity matrices can be expressed as functions $\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}}, \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{\theta}} : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}^{n \times n}$ of the form

$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}}(\boldsymbol{v}, \boldsymbol{q}) = \operatorname{diag}(\boldsymbol{v}) \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{v}} - 2 \operatorname{diag}(\boldsymbol{q}), \tag{11a}$$

$$\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{\theta}}(\boldsymbol{v}, \boldsymbol{p}) = -\operatorname{diag}(\boldsymbol{v}) \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{v}} + 2 \operatorname{diag}(\boldsymbol{p}), \tag{11b}$$

which are implicitly parameterized by $\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{v}}$ and $\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{v}}$.

# New characterization of the structure of the Power Flow Jacobian

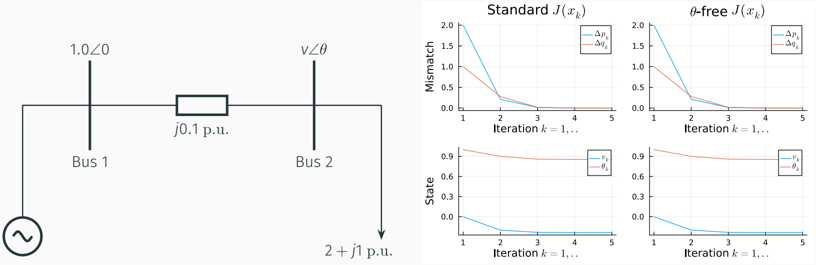


**Figure 8:** Equivalence of the Newton-Raphson iterations using the $\theta$-free expressions and standard expressions for $\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}}, \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{\theta}}$ (right) for a simple two bus test case (left).

# Phase retrieval by the power flow Jacobian

Apply a classic phase retrieval algorithm [4] to solve the power flow equations using learned voltage magnitude blocks

$$\underset{\Delta\boldsymbol{\theta}_t,}{\text{minimize}} \quad \left\| \begin{bmatrix} \Delta \boldsymbol{p}_t \\ \Delta \boldsymbol{q}_t \end{bmatrix} - \begin{bmatrix} (?) & \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{v}} \\ (?) & \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{v}} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta}_t \\ \Delta \boldsymbol{v}_t \end{bmatrix} \right\|_2^2$$

Learning the Jacobian blocks is well studied [5].

# Phase retrieval by the power flow Jacobian

The phase retrieval program for samples $t = 1, \ldots,$ can then be written as

$$\begin{aligned} \underset{\Delta\boldsymbol{\theta}_t, \frac{\partial \mathbf{p}}{\partial \boldsymbol{\theta}}, \frac{\partial \mathbf{q}}{\partial \boldsymbol{\theta}}}{\text{minimize}} \quad & \left\| \begin{bmatrix} \Delta \mathbf{p}_t \\ \Delta \mathbf{q}_t \end{bmatrix} - \begin{bmatrix} \frac{\partial \mathbf{p}}{\partial \boldsymbol{\theta}}(\mathbf{v}_t, \mathbf{q}_t) & \frac{\partial \mathbf{p}}{\partial \mathbf{v}} \\ \frac{\partial \mathbf{q}}{\partial \boldsymbol{\theta}}(\mathbf{v}_t, \mathbf{p}_t) & \frac{\partial \mathbf{q}}{\partial \mathbf{v}} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta}_t \\ \Delta \mathbf{v}_t \end{bmatrix} \right\|_2^2 \\ \text{subject to:} \quad & \text{power flow Jacobian structure!} \end{aligned}$$

Use the power flow Jacobian structure to guarantee voltage phase retrieval

# Guaranteed phase retrieval via spectral theory

Long story short:

1. Use the Jacobian structure to show when the voltage phase can be uniquely recovered.
2. Use spectral theory to bound the eigenvalues of the full Jacobian using the phaseless expressions

# Guaranteed phase retrieval via spectral theory

**Theorem (Phase retrieval from active power injections)**

*For a set of buses in a network $\mathcal{B} \subset \{1, \ldots, n\}$, if for every bus $i \in \mathcal{B}$, the reactive power differential inequality*

$$|q_i| > \frac{1}{2} v_i \left( \sum_{k \in \mathcal{B} \setminus \{i\}} \left| \frac{\partial q_k}{\partial v_i} \right| - \left| \frac{\partial q_i}{\partial v_i} \right| \right) \tag{12a}$$

$$\text{or} \quad |q_i| > \frac{1}{2} \left( \sum_{k \in \mathcal{B} \setminus \{i\}} v_k \left| \frac{\partial q_i}{\partial v_k} \right| - v_i \left| \frac{\partial q_i}{\partial v_i} \right| \right), \tag{12b}$$

*holds, then the voltage phase angles can be uniquely recovered from solely the active power (real) injections* $\mathbf{p}$.

# Guaranteed phase retrieval via spectral theory

**Theorem (Phase retrieval from reactive power injections)**

*Analogously, if for every bus $i \in \mathcal{B}$, the active power differential inequality*

$$|p_i| > \frac{1}{2} v_i \left( \sum_{k \in \mathcal{B} \setminus \{i\}} \left| \frac{\partial p_k}{\partial v_i} \right| - \left| \frac{\partial p_i}{\partial v_i} \right| \right) \tag{13a}$$

$$\text{or} \quad |p_i| > \frac{1}{2} \left( \sum_{k \in \mathcal{B} \setminus \{i\}} v_k \left| \frac{\partial p_i}{\partial v_k} \right| - v_i \left| \frac{\partial p_i}{\partial v_i} \right| \right), \tag{13b}$$

*holds, then the voltage phase angles can be uniquely recovered from solely the reactive power (imaginary) injections $\boldsymbol{q}$.*

# Jacobian invertibility guarantees

Side note: can also guarantee Jacobian invertibility

Jacobian invertibility $\iff$ No voltage collapse $\implies$ Phase retrieval

Take a look at the paper for more details

# Computational Results

1. RTS-GMLC network model [6]
2. Open-source
3. Real-world data

How does this compare to classical model-based state estimation?

# Comparison with classical state estimation

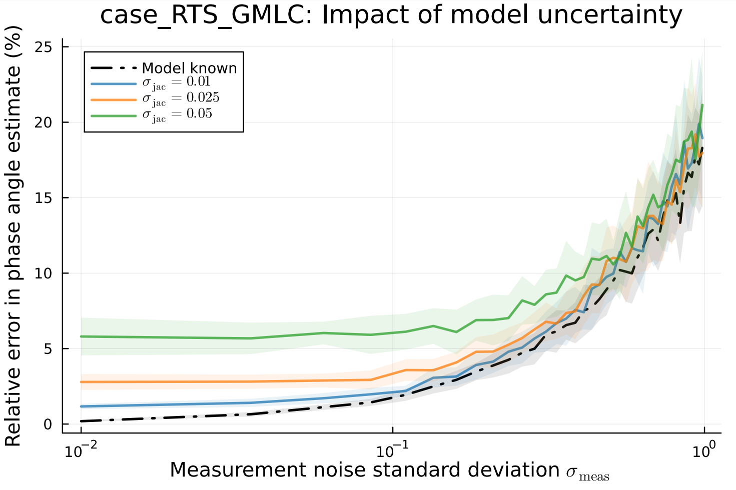


**Figure 9:** Impact of (*RTS_GMLC*) model uncertainty on recovered phase angle relative error vs. measurement noise level. Shaded regions indicate ±1 standard deviation of the relative errors computed over 20 bootstraps.

# Voltage phasor recovery performance

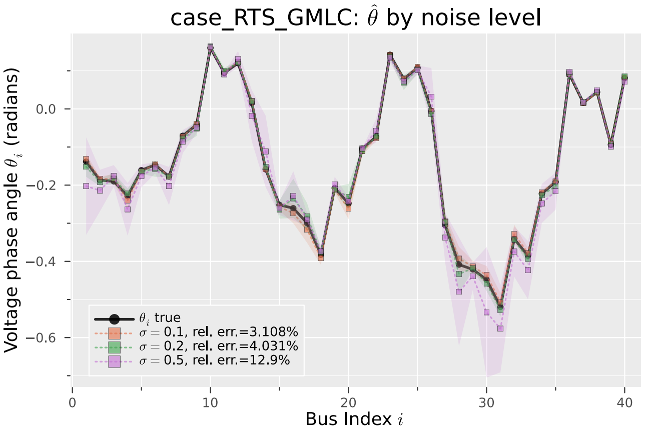


Figure 10: Voltage phasor recovery by measurement noise level.

How about the matrices?

# Power flow Jacobian recovery

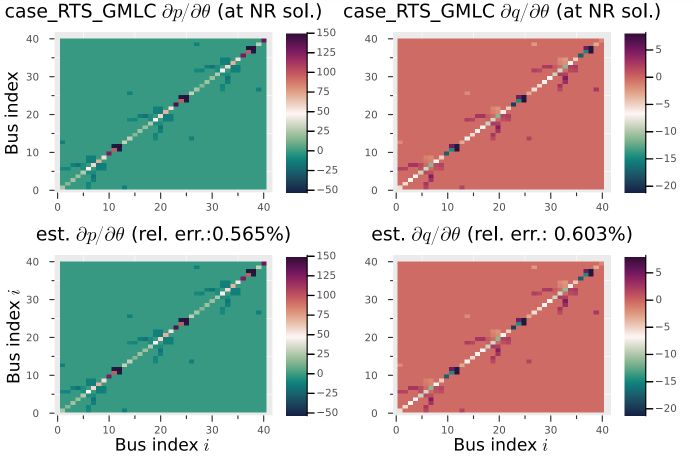


Figure 11: Recovery of the power-phase angle submatrices $\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}}, \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{\theta}}$ of the power flow Jacobian for the *RTS_GMLC* network via the phase retrieval program.

How about real-time performance?

# Real-time voltage phasor recovery performance

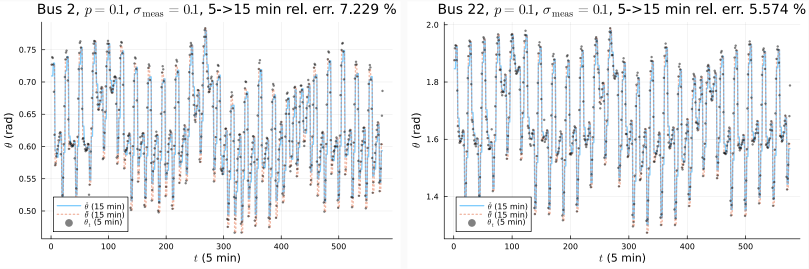


**Figure 12:** Ground truth (blue) and estimated (orange dashed) voltage phase angles at 15 min. granularity, juxtaposed with ground truth 5 min. granularity voltage phase angles (black dots).

# Sensitivity matrix recovery

**Inverse problem: voltage magnitude sensitivity matrix recovery**

What is the causation (voltage magnitude-complex power sensitivity matrices) of our voltage magnitude measurements?

$$\boldsymbol{v} = F\left(\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{p}}^*, \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{q}}^*\right) + \boldsymbol{\eta}$$

# Voltage magnitude Taylor series expansion.

Construct an underdetermined linearization of the voltage magnitudes via Taylor Series:

**Definition: Voltage magnitude linearization**

$$\underbrace{\Delta v}_{(|\mathcal{N}|\times 1)} = \underbrace{\begin{bmatrix} \frac{\partial v}{\partial p} & \frac{\partial v}{\partial q} \end{bmatrix}}_{(|\mathcal{N}|\times 2|\mathcal{N}|)} \underbrace{\begin{bmatrix} \Delta p \\ \Delta q \end{bmatrix}}_{(2|\mathcal{N}|\times 1)} = \underbrace{|Sx|}_{(|\mathcal{N}|\times 1)} . \tag{14}$$

# Representing reactive power as an equivalent active power

Given power factors $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_n]^T \in (0,1]^n$ of the net complex power injections at each bus, there exists a matrix function $K(\boldsymbol{\alpha}) : (0,1]^n \mapsto \mathbb{R}^{n \times n}$ of the form

$$K(\boldsymbol{\alpha}) = \begin{bmatrix} \pm\frac{1}{\alpha_1}\sqrt{1-\alpha_1^2} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \pm\frac{1}{\alpha_n}\sqrt{1-\alpha_n^2} \end{bmatrix}, \tag{15}$$

such that the reactive power can be written as a parameterized function of the active power [7]

**$q$ as a function of $p$ given $\alpha$**

$$q(p|\boldsymbol{\alpha}) = K(\boldsymbol{\alpha})p. \tag{16}$$

# Graphical Example

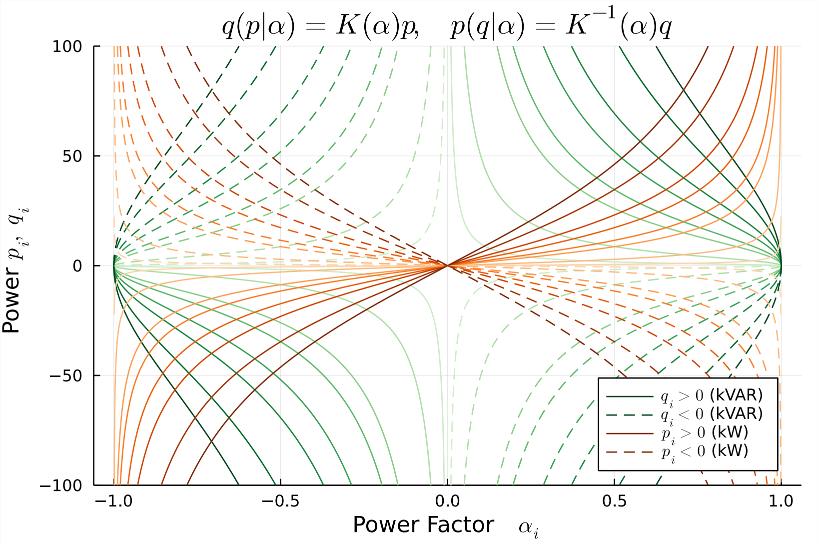


**Figure 13:** Representing $q$ and $p$ as parameterized function of $\alpha$

# Phaseless sensitivity operator

Use $K$ to construct an equivalent square system.

**Definition: phaseless voltage sensitivity operator**

$$
\begin{aligned}
\underbrace{\Delta v}_{(|\mathcal{N}|\times 1)} &= \underbrace{\begin{bmatrix} \frac{\partial v}{\partial p} & \frac{\partial v}{\partial q} \end{bmatrix}}_{(|\mathcal{N}|\times 2|\mathcal{N}|)} \underbrace{\begin{bmatrix} \Delta p \\ \Delta q \end{bmatrix}}_{(2|\mathcal{N}|\times 1)} = \underbrace{|Sx|}_{(|\mathcal{N}|\times 1)} \\
&= \underbrace{\left( \frac{\partial v}{\partial p} + \frac{\partial v}{\partial q} K(\alpha) \right)}_{|\mathcal{N}|\times|\mathcal{N}|} \underbrace{\Delta p}_{|\mathcal{N}|} \\
&\triangleq S(\alpha)\Delta p.
\end{aligned}
$$

Solution: show when $S(\alpha)$ is invertible

# Invertibility of phaseless sensitivity operator

**Theorem: phaseless observability**

Let $\Delta K := \max_{i \in \mathcal{N}} K_{i,i} - \mathsf{I}$. Then the complex power injections can be estimated from the voltage magnitudes if

$$\left\| \mathsf{M}^{-1} \Delta \mathsf{K} \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}} \right\|_2 < 1, \tag{17}$$

where $\|\cdot\|_2$ is the largest singular value—the spectral norm or operator norm—of the argument.

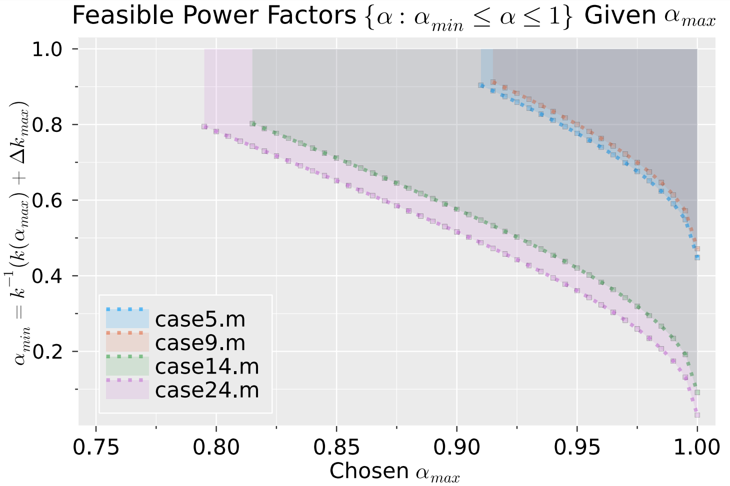


**Figure 14:** Feasible power factors such that the phaseless observability Theorem holds for several popular test cases.

# Outlook

# What do we have?

We have shown that we can:

1. Recover voltage phasors from their magnitudes
2. Recover complex power injections from voltage magnitudes
3. Recover models of voltage phase angles from their magnitudes
4. Recover the power flow Jacobian blocks and the blocks of its inverse

This can save a lot of money[4]

[4] A PMU installation costs $40,000-$180,000 each.
"Factors affecting PMU installation costs", US Department of Energy, 2014.

# What are the limitations?

1. Bottle necked by measurement frequency (PMUs, milliseconds, other devices, minutes-hours)
2. Bottle necked by measurement type (Reactive power assumptions needed)
3. Needs further incorporation of the non-linearities of the AC power flow equations.

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Thanks for listening

- M. Kezunovic, “Panel session VII: PMU testing and synchrophasor system life-cycle management,” in *International Conference on Smart Grid Synchronized Measurements and Analytics (SGSMA)*. Split, Croatia: IEEE Power and Energy Society and IEEE Instrumentation and Measurement Society, May 2022.
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# Backup slides

# Background: Gershgorin Circle Theorem

**Where are the eigenvalues of a square matrix?**

For any matrix $\boldsymbol{A} \in \mathbb{C}^{n \times n}$, by the ***Gershgorin Circle Theorem*** the eigenvalues of $\boldsymbol{A}$ are guaranteed to lie in the union of the $i = 1, \ldots, n$ Gershgorin discs $\mathcal{G}_i(\boldsymbol{A})$ of the matrix, i.e.,

$$\lambda_i(\boldsymbol{A}) \in \bigcup_{i=1}^{n} \mathcal{G}_i(\boldsymbol{A}), \quad i = 1, \ldots, n, \tag{18}$$

where

$$\mathcal{G}_i(\boldsymbol{A}) \triangleq \{w \in \mathbb{C} : |w - A_{ii}| \leq \sum_{k: k \neq i} |A_{ik}|\} \subseteq \mathbb{C}. \tag{19}$$

# Classical phase retrieval

**Given:** $|Ax| = b$, $\boxed{b \in \mathbb{R}^m,}$ what is $x \in \mathbb{C}^n$?

**Classical phase retrieval problem:**

$$\min_{x\in\mathbb{C}^n, y\in\mathbb{C}^m} ||Ax - y||_2^2 \quad \text{s.t.} \quad |y| = b, \tag{20a}$$

$$\iff \min_{x\in\mathbb{C}^n, u\in\mathbb{C}^m} ||Ax - \operatorname{diag}(b)u|| \quad \text{s.t.} \quad |u| = \mathbb{1}, \tag{20b}$$

$$\iff \min_{u\in\mathbb{C}^m : |u_i|=1 \ \forall i \in [\![1,m]\!]} u^* M u, \quad \text{s.t.} \quad M = \operatorname{diag}(b - AA^\dagger) \succ 0. \tag{20c}$$

for any candidate phase $u'$, note that

$$\boxed{\hat{x} = A^\dagger \operatorname{diag}(b)u' = (A^*A)^{-1}A^* \operatorname{diag}(b)u',} \tag{21}$$

# Theorem accuracy

| Case | # PQ Buses | % Satisfying Thm. 1 | $r_{\text{worst}}$ |
|---|---|---|---|
| *14* | 9 | 100.0% | — |
| *24_ieee_rts* | 13 | 100.0% | — |
| *ieee30* | 24 | 95.83% | $1.4 \times 10^{-14}$ |
| *RTS_GMLC* | 40 | 100.0% | — |
| *118* | 64 | 100.0% | — |
| *89pegase* | 77 | 94.81% | 8.32 |
| *ACTIVSg200* | 162 | 96.91% | 0.088 |
| *ACTIVSg500* | 444 | 94.37% | 3.014 |
| *ACTIVSg2000* | 1608 | 84.83% | 28.31 |

Table 2: Analysis of Theorem 1 for PQ buses of various test cases

# Theorem accuracy

| Case | # PQ Buses | % Satisfying Thm. ?? | $\sigma_{\max}$ |
|---|---|---|---|
| *14* | 9 | 100.0% | 0.876 |
| *24_ieee_rts* | 13 | 100.0% | 0.401 |
| *ieee30* | 24 | 95.83% | 1.437 |
| *RTS_GMLC* | 40 | 100.0% | 0.444 |
| *118* | 64 | 100.0% | 0.473 |
| *89pegase* | 77 | 100% | 0.954 |
| *ACTIVSg200* | 162 | 100% | 0.698 |
| *ACTIVSg500* | 444 | 99.77% | 1.090 |
| *ACTIVSg2000* | 1608 | 99.69% | 1.180 |

**Table 3:** Analysis of Theorem **??** for PQ buses of various test cases

# Distribution networks

| Quantity | Value |
| --- | --- |
| $\left\Vert \frac{\partial p}{\partial \boldsymbol{\theta}} - \frac{\partial p}{\partial \boldsymbol{\theta}}(v, q) \right\Vert_F \Big/ \left\Vert \frac{\partial p}{\partial \boldsymbol{\theta}} \right\Vert_F$ | $2.510 \times 10^{-8}$ |
| $\left\Vert \frac{\partial q}{\partial \boldsymbol{\theta}} - \frac{\partial q}{\partial \boldsymbol{\theta}}(v, p) \right\Vert_F \Big/ \left\Vert \frac{\partial q}{\partial \boldsymbol{\theta}} \right\Vert_F$ | $1.725 \times 10^{-7}$ |

**Table 4:** Verification that the structure expressions of the Lemma hold for multiphase unbalanced networks.